Landauer approach to time-dependent transport

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Abstract

Based upon the nonequilibrium Green's function formalism, we present a time-dependent Landauer approach to transport through a mesoscopic system under an ac bias voltage. The system is modeled as an elastic scatterer coupled to large electron reservoirs through perfect conducting wires (leads). The chemical potentials of the reservoirs are driven apart by the bias and, consequently, current flows through the leads from one reservoir to another. We examine the nonequilibrium statistical processes of electrons in the leads. The electronic waves are quantized on the basis of orthonormal wave packets moving along the leads, scattered by the scatterer, and coupled to the reservoirs. The time for an electron to traverse the leads between the source and the drain reservoirs plus the phase delay time caused by the scatterer is found to be the relevant time scale in the time-dependent transport. The frequency dependence of the admittance is fully investigated.

73.40.-c,72.10.Bg,85.30.De

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I. INTRODUCTION

Nano-fabrication technology has brought great interest in and enabled thorough investigation into mesoscopic systems in which quantum coherence nature of electrons dominates the transport characteristics. [1,2] A prototype example is the double-barrier resonant-tunneling structure which exhibits unique transport characteristics. [3–9] Another example mesoscopic system is illustrated in Fig.1. It consists of an elastic scatterer and two or more perfect conducting leads connecting the scatterer to the reservoirs. Throughout the lead-scatterer region, the electronic waves are kept phase-coherent. Strong phase-breaking scattering in a reservoir keep electrons there being in local equilibrium with the reservoir chemical potential. When the reservoir chemical potentials are driven apart from each other by an external voltage bias, current flows from one reservoir to another in response. The dc conductance matrix is directly related to the scattering matrix by the Landauer approach. [10–13] The goal of this paper is to study the frequency response of mesoscopic systems biased with ac voltage sources. Based upon the nonequilibrium Green’s function formalism, a time-dependent Landauer approach will be carried out to the frequency-dependent admittance.

While the dc characteristics of various mesoscopic systems are well understood, investigation of dynamic properties has been rather limited. In double-barrier resonant-tunneling structures, the energy scale $\Delta \varepsilon$ over which the transmission coefficient $|t|^2$ varies significantly determines the characteristic frequency and the transient time [4–9] since the time scale $\tau = \hbar/\Delta \varepsilon$ is the longest among all the relevant time scales. When the frequency of an ac bias is greater than $2\pi/\tau$, the system becomes unable to respond instantaneously and, consequently, the conductance falls below its dc level. The ac transport in mesoscopic systems such as shown Figs.1 and 2 has been studied in two recent papers. [14,15] In Ref.14, of the phase shift in the scattering matrix gives a relevant time scale for the frequency-dependent admittance. In a perfect tow-probe system without the center scatterer, however, the con-
ducting channel is completely open and the transmission coefficient is energy independent. Then the aforementioned time scale as determined by the energy dependence of the scattering matrix does not exist and one faces the question what is the relevant transient time scale. Ref. 15 answered this question and provided a derivation of the frequency dependent conductance as well as the imaginary part of the admittance. For such a completely open channel system, the relevant time scale was found to be determined by the time for an electron to traverse the conducting channel from the source reservoir to the drain, namely, \( \tau_0 = L/v_g \) where \( v_g \) is the group velocity of an electron wave packet in the channel and \( L \), length of the conducting channel. In a general mesoscopic system, both aspects, i.e., the scattering delay and the traversing time, should be taken into account on a consistent basis.

In this paper, we present a time-dependent Landauer approach to the dynamic transport through mesoscopic systems which is based upon the nonequilibrium Green’s function formalism. We take a two-probe mesoscopic system illustrated in Fig. 2 as the working example to fully carry out the calculation but the formalism is valid for multi-probe systems in general. The system consists of two reservoirs at chemical potential \( \mu_1 \) and \( \mu_2 \) and two perfect conducting leads connecting the scatterer in the center to the two reservoirs. The reservoirs are assumed to be dissipative so that electrons there are always in local equilibrium even when the whole system is driven out of equilibrium by an external bias. The leads and scatterer are assumed dissipationless and the lead-reservoir interfaces, reflectionless. A wave emanating from reservoir 1 (or 2) moves through lead 1 (2) and gets scattered in the center. Part of the wave, scattered forward, continues to move through lead 2 (1) and enters into reservoir 2 (1) without reflection. The other part of the wave, reflected, moves back along lead 1 (2) and enters reservoir 1 (2) without reflection. When the system is biased with an external ac voltage \( u(t) \), the reservoir chemical potentials are driven apart,

\[
\mu_1 - \mu_2 = eu(t) = e\alpha e^{-i\omega t}. \tag{1}
\]

In response, a time-dependent current \( i(t) \) flows through the leads between the reservoirs. In the linear response regime,
where \( Y(\Omega) \) is the frequency dependent admittance. In order to calculate \( Y(\Omega) \) and \( i(t) \), in general, we examine the nonequilibrium statistical processes of electrons in the leads subject to scattering over the scatterer and subject to coupling with the reservoirs. The electronic waves are quantized on the basis of orthonormal wave packets moving along the leads, which are composed from scattering plane waves. The relevant time scale is found to be the time for an electron to traverse the leads between the source and the drain reservoirs plus the phase delay time caused by the scatterer

\[
\tau = \frac{L}{v_g} + \frac{\hbar}{dE}
\]

where \( \delta(E) \) is the phase-shift of the scattered wave function with energy \( E \). The characteristic frequency of the ac conductance \( \Omega_0 = 2\pi/\tau \). With increasing frequency \( \Omega \) of the applied bias voltage, the response current decreases and its spatial variation at length scale of \( l_\Omega = v_g/\Omega \) also becomes observable at high frequencies. In section II, the Landauer approach to \( dc \) transport is reviewed and a time dependent wave packet formalism is proposed. In section III, the time dependent current is calculated and the frequency-dependent admittance is derived. In section IV is a summary.

II. TIME DEPENDENT WAVE PACKET FORMALISM

In the Landauer approach, the scattering wave functions are of essential importance. [10] We choose the coordinates with \( x \)-axis along the leads from reservoir 1 to reservoir 2 with the scatterer at \( x=0 \). Reservoirs 1 and 2 at \( x \leq -L/2 \) and \( x \geq L/2 \) respectively. The scattering plane wave function incident from the left and that from the right are, respectively,

\[
\psi_{k,L}(x,t) = e^{-iEt/\hbar} \begin{cases} 
 e^{ikx} + r(k)e^{-ikx} & x < 0 \\
 t(k)e^{ikx} & x > 0 
\end{cases}
\]

(4)
\[
\psi_{k,R}(x, t) = e^{-iEt/\hbar} \begin{cases} 
    t(k)e^{-i{k}x} & x < 0 \\
    e^{-i{k}x} + r(k)e^{i{k}x} & x > 0
\end{cases},
\]
where \( E = \hbar^2 k^2 / 2m \) is the energy. \( t(k) = |t|e^{i\delta(E)} \) and \( r(k) = |r|e^{i\delta_r(E)} \) are respectively the transmission and reflection coefficient. When the system is biased with a dc voltage (\( \Omega = 0 \)), \( \psi_{k,L}(x, t) \)–state is assumed to be occupied with the Fermi-Dirac distribution \( f(E, \mu_1) \) with chemical potential \( \mu_1 \) while \( \psi_{k,R}(x, t) \)–state is assumed to be occupied with the Fermi-Dirac distribution \( f(E, \mu_2) \) with chemical potential \( \mu_2 \). Since \( \psi_{k,L}(x, t) \) carries a current \(-e\hbar k|t|^2/m\) and \( \psi_{k,R}(x, t) \) carries \( e\hbar k|t|^2/m\), the total current flowing from reservoir 1 to 2 is
\[
I = \frac{1}{L_0} \sum_k |t|^2 \frac{e\hbar k}{m} (f(E, \mu_2) - f(E, \mu_1)).
\]  
Here \( L_0 \) is the size of the entire system. The conductance at zero temperature can be found readily,
\[
G = Y(\Omega = 0) = |t|^2 \frac{e^2}{\hbar}.
\]  
When the system is biased with an ac voltage \( u(t) \), the occupations on the scattering wave functions are not known \textit{apriori}. They have to be determined by the quantum statistical mechanics. Therefore, second quantization is necessary for deriving the ac transport coefficient. Ref.14 performed the second quantization with the plane waves as the orthonormal basis set while assuming the occupation on \( \psi_{k,L}(x, t) \) being \( f(E, \mu_1(t)) \) and that on \( \psi_{k,R}(x, t) \) being \( f(E, \mu_2(t)) \). Apparently, this procedure is not self-consistent. Furthermore, the plane waves are simply not appropriate for performing the required second quantization because the scattering plane waves, although complete, are not mutually orthogonal. Moreover, a plane wave, extending throughout the leads and scatterer from reservoir 1 to reservoir 2, is necessarily coupled to both reservoirs at any time. This simultaneous coupling with both reservoirs makes self-consistent statistics on plane waves practically intractable. These problems can be solved with a complete set of orthonormal wave packets formed from linear combinations of scattering plane waves. The wave packets \( \Psi_{k,x_0}^L(x, t) \) incident from the left and \( \Psi_{k,x_0}^R(x, t) \) incident from the right are found as follows. [16]
\[ \Psi_{k,x_0}^L (x,t) = e^{-iEt/\hbar} \begin{cases} 
eq e^{ikx} \phi_\xi (x-x_0-v_k t) & x < 0, \\
+ r(k)e^{-ikx} \phi_\xi (x+x_0+v_k (t - \tau_s)) & x > 0 \end{cases} \]

\[ \Psi_{k,x_0}^R (x,t) = e^{-iEt/\hbar} \begin{cases} 
eq t(k)e^{ikx} \phi_\xi (x-x_0-v_k (t - \tau_s)) & x < 0, \\
- e^{ikx} \phi_\xi (x-x_0-v_k t) & x = 0, \\
+ r(k)e^{ikx} \phi_\xi (x+x_0-v_k (t - \tau_s)) & x > 0 \end{cases} \]

where \( \tau_s = d\delta_t/dE = d\delta_x/dE \) is the phase-delay time caused by scattering and \( v_k = \hbar k/m \).

The packet function

\[ \phi_{k,x_0} (x) = e^{ik(x-x_0)} \frac{1}{L_0} \sum_{q=-\sigma/2}^{\sigma/2} e^{iq(x-x_0)} = e^{ikx} \phi_\xi (x-x_0) \]

is centered at \( x_0 \) with a width \( \xi = 2\pi/\sigma \). The wave packets thus constructed form an orthonormal set if we choose \( k \) to be multiples of \( \sigma \) \((k >> \sigma)\) and \( x_0 \) to be multiples of \( \xi \). The reservoir wave functions \( \phi_{p}^L (x,y) \) and \( \phi_{p}^R (x,y) \) and the scattering wave packets \( \{ \Psi_{k,x_0}^L (x,t), \Psi_{k,x_0}^R (x,t) \} \) together form a complete orthonormal set. Based on this set of wave functions, the second quantization of electron waves can be performed. The electron field operator

\[ \psi(x,y,t) = \sum_{k,x_0} c_{k,x_0}^L (t) \Psi_{k,x_0}^L (x,t) + c_{k,x_0}^R (t) \Psi_{k,x_0}^R (x,t) \]

\[ + \sum_p a_p \phi_p^L (x,y) + \sum_p b_p \phi_p^R (x,y) \]

and its Hermitian conjugate \( \psi^\dagger (x,y,t) \) satisfy the canonical anti-commutation relations and \( c_{k,x_0}^{L(R)} \) and \( c_{k,x_0}^{L(R)^\dagger} \) are, respectively, the canonical fermionic annihilation and creation operators on the state \( \Psi_{k,x_0}^{L(R)} (x,t) \). \( a_p, a_p^\dagger, b_p, \) and \( b_p^\dagger \) are, respectively, the annihilation and creation operators on the reservoir wave function \( \phi_{p}^L (x,y) \) and \( \phi_{p}^R (x,y) \) states.

Obviously, there exist couplings between the scattering wave packets and the reservoir states by, for example, impurities in reservoir regions adjacent to the leads. This coupling
can be modeled in a simple transfer matrix form. Then the Hamiltonian of the system can be written as

\[
H = \sum_{k,x_0} E_k \left( c_{k,x_0}^L(t)c_{k,x_0}^L(t) + c_{k,x_0}^R(t)c_{k,x_0}^R(t) \right) \\
+ H_1(a, a^\dagger) + H_2(b, b^\dagger) + H'_L(t) + H'_R(t) \tag{12}
\]

where \( H_1 \) and \( H_2 \) are the Hamiltonians of the two reservoirs. \( H'_L \) and \( H'_R \) are the coupling between the scattering wave packets and the two reservoirs.

\[
H'_L(t) = \sum_{k,x_0} a_p^\dagger(t)c_{k,x_0}^L(t)T_{kp}^L(-x_0 + v_k t) + h.c.
+ \sum_{k,x_0} a_p(t)d_{k,x_0}(t)T_{kp}^L(x_0 - v_k t - \tau_s) + h.c., \tag{13}
\]

\[
H'_R(t) = \sum_{k,x_0} b_p^\dagger(t)c_{k,x_0}^R(t)T_{kp}^R(x_0 - v_k t) + h.c.
+ \sum_{k,x_0} b_p(t)e_{k,x_0}(t)T_{kp}^R(-x_0 + v_k t - \tau_s) + h.c., \tag{14}
\]

where

\[
d_{k,x_0}(t) = c_{k,x_0}^L(t)r(k) + c_{k,x_0}^R(t)t(k), \tag{15}
\]

\[
e_{k,x_0}(t) = c_{k,x_0}^L(t)t(k) + c_{k,x_0}^R(t)r(k), \tag{16}
\]

and their Hermitian conjugates also satisfy the canonical anti-commutation relations. The transfer matrices can be written as

\[
T_{kp}^L(u) = \int dy \int_{-\infty}^{L/2} dx \phi_p^{L*}(x, y)T(x, y)e^{i\xi x\phi_{L}(x - u)}, \tag{17}
\]

\[
T_{kp}^R(u) = \int dy \int_{L/2}^{\infty} dx \phi_p^{R*}(x, y)T(x, y)e^{-i\xi x\phi_{L}(x - u)}. \tag{18}
\]

The first term in Eq.(13) represents the coupling of the incident wave packets with the left reservoir and the second term is the coupling of the scattering-out wave packets with the left reservoir. Likewise, Eq.(14) is the coupling of the incident and the scattering-out wave packets with the right reservoir. From Eqs.(17) and (18), we note that, for a wave packet with momentum \( \hbar k \) and centered at \( v_k t - x_0 \), only one of the two types of coupling to a reservoir is not vanishing at a give time \( t \) (either the incident part or the scattering-out part) since \( L >> \xi \).
III. FREQUENCY DEPENDENT ADMITTANCE

When the length of the leads $L >> \xi$, the current carried by hopping from one reservoir to the other via a channel state is negligibly small and charges are only transported by the drifting motion of the wave packets. Namely, the current operator can be written as

$$i(x,t) = \begin{cases} 
\sum_{k,x_0} c^\dagger_{k,x_0}(t)c_{k,x_0}(t) \frac{\gamma_k}{m} |\phi_k(x + x_0 - v_k t)|^2 & x < 0 \\
- \sum_{k,x_0} d^\dagger_{k,x_0}(t)d_{k,x_0}(t) \frac{\gamma_k}{m} |\phi_k(x - x_0 + v_k(t - \tau_s))|^2 & x > 0 
\end{cases}$$

(19)

whose quantum statistical average yields the intrinsic current. Generally, the measured current, i.e., the circuit current $I(t)$ can be a complicated convolution of $\langle i(x,t) \rangle$ which involves the detailed information of the conductor geometry, the voltage probe contacts and, especially, the capacitance distribution. [10] In what follows we shall concentrate our discussion on the intrinsic part with emphasis on the physics involved.

Due to strong dissipation in the reservoirs, electrons there can be well described by separate equilibrium density matrices with chemical potentials $\mu_1(t)$ and $\mu_2(t)$ respectively, which are driven away from the Fermi energy $E_F$ by the applied bias voltage $u(t)$: $\mu_1 = E_F + eu(t)$ and $\mu_2 = E_F$. With help of the Keldysh formalism of nonequilibrium Green’s functions, [17] the following occupation numbers can be found.

$$\langle c^\dagger_{k,x_0}(t)c_{k,x_0}(t) \rangle = n_L e^{-\int_{-\infty}^t d\tau_0 \gamma_0(k,x_0,\tau_0)} + |r(k)|^2 n_d + |t(k)|^2 n_e,$$

(20)

$$\langle c^\dagger_{k,x_0}(t)c_{k,x_0}(t) \rangle = n_R e^{-\int_{-\infty}^t d\tau_0 \gamma_0(k,x_0,\tau_0)} + |t(k)|^2 n_d + |r(k)|^2 n_e,$$

(21)

$$\langle d^\dagger_{k,x_0}(t)d_{k,x_0}(t) \rangle = n_d + \left( |r(k)|^2 n_L + |t(k)|^2 n_R \right) e^{-\int_{-\infty}^t d\tau_0 \gamma_0(k,x_0,\tau_0)},$$

(22)

$$\langle c^\dagger_{k,x_0}(t)c_{k,x_0}(t) \rangle = n_e + \left( |t(k)|^2 n_L + |r(k)|^2 n_R \right) e^{-\int_{-\infty}^t d\tau_0 \gamma_0(k,x_0,\tau_0)},$$

(23)

with
\begin{align}
   n_L(k, x_0, t) &= \int_{-\infty}^{t} du e^{-\int_{u}^{t} d\tau \gamma_1(k, x_0, \tau)} \gamma_1(k, x_0, u) f_1(u), \quad (24) \\
   n_R(k, x_0, t) &= \int_{-\infty}^{t} du e^{-\int_{u}^{t} d\tau \gamma_2(k, x_0, \tau)} \gamma_2(k, x_0, u) f_2(u), \quad (25) \\
   n_d(k, x_0, t) &= \int_{-\infty}^{t} du e^{-\int_{u}^{t} d\tau \gamma_3(k, x_0, \tau)} \gamma_3(k, x_0, u) f_1(u), \quad (26) \\
   n_e(k, x_0, t) &= \int_{-\infty}^{t} du e^{-\int_{u}^{t} d\tau \gamma_4(k, x_0, \tau)} \gamma_4(k, x_0, u) f_2(u), \quad (27)
\end{align}

where \( f_{1(2)}(t) = f(E_k, \mu_{1(2)}(t)) \). \( \gamma_i(k, x_0, t) \) is coupling rate of the wave packet with one of the reservoirs. The detailed evaluation of \( \gamma_i \) requires precise knowledge of the lead-reservoir interfaces as well as the scattering mechanisms. From the coupling Hamiltonian \( H'_L \) and \( H'_R \), they can be written in the following form with the overall scattering rate \( \gamma \) as a parameter.

\begin{align}
   \gamma_1(k, x_0, t) &= \gamma \int_{-L/2}^{L/2} dx |\phi_\xi(x + x_0 - v_k t)|^2, \quad (28) \\
   \gamma_2(k, x_0, t) &= \gamma \int_{L/2}^{L/2} dx |\phi_\xi(x + x_0 - v_k t)|^2, \quad (29) \\
   \gamma_3(k, x_0, t) &= \gamma \int_{-L/2}^{L/2} dx |\phi_\xi(x - x_0 + v_k(t - \tau_s))|^2, \quad (30) \\
   \gamma_4(k, x_0, t) &= \gamma \int_{L/2}^{L/2} dx |\phi_\xi(x + x_0 - v_k(t - \tau_s))|^2. \quad (31)
\end{align}

The final results will be independent of \( \gamma \) as long as \( L \gg \xi \). In Fig. 3, we illustrate the time-dependence of \( \gamma_{1(2)} \). Note that \( \gamma_1(k, x_0, t) \) evolves with time monotonously from \( \gamma \) where \( (v_k t - x_0) \ll -L/2 \) to 0 where \( (v_k t - x_0) \gg -L/2 \). Moreover, the variation is mainly located around \( (v_k t - x_0) = -L/2 \). Similarly, \( \gamma_2(k, x_0, t) \) evolves from 0 at \( (v_k t - x_0) \ll L/2 \) to \( \gamma \) at \( (v_k t - x_0) \gg L/2 \). Parallel behaviors are observed for \( \gamma_3 \) and \( \gamma_4 \). Inside the leads, all the \( \gamma_i \)’s clearly vanish.

Examining the integrands in Eqs. (24)-(27), we can see the origin of the relevant time of the system. For example, following an electron (a wave packet) emanating from the left reservoir into the lead at time \( t = 0 \), it moves ballistically in the dissipationless lead 1 with definite velocity \( v_k = \hbar k/m \). When it arrives at the center at \( t \approx L/2v_k \), it is scattered by the scatterer and two scattering-out wave packets emerge at \( t \approx L/2v_k + \tau_s \). The scattering-out wave packets move at \( v_k \) along leads 1 and 2 until they reach the lead-reservoir interfaces at \( t \approx L/v_k + \tau_s \). Apparently, after the electron gets into lead 1 and before the emerging
scattering-out packets arrive at the two lead-reservoir interfaces, it is free from influence of
the changes in the two reservoirs. The time interval during which the reservoirs are unable
to influence an electron is \( \tau = L/v_k + \tau_s \). Consequently, the system is unable to respond to
an applied bias on a time scale shorter than \( \tau \).

Now we study the frequency dependent admittance of the system subject to an applied
ac bias \( u(t) = u_0 e^{-i\Omega t} \). When the signal voltage \( |u(t)| \ll E_F/e \), the reservoir distribution
functions can be expanded as

\[
  f_1(t) = f_0 + f'_0(E_k) e u(t), \quad f_2 = f_0
\]

where \( f_0(E_k) \) is the equilibrium Fermi-Dirac distribution function. Then the current becomes
\( \langle i(x,t) \rangle = Y(x,\Omega) u(t) \). At zero temperature, \( f'_0(E_k) = -\delta(E_k - E_F) \), the admittance
\( Y(x,\Omega) = G y(x,\Omega) \) with

\[
y(x,\Omega) = \begin{cases} \int_{-\infty}^{t} du e^{i\Omega(t-u)} \sum_{k,x_0} \left[ e^{-\int_{u}^{t} d\tau \gamma_1(k,x_0,\tau)} \gamma_1(k,x_0,\tau) \times \\
|\phi_\xi(x + x_0 - v_gt)|^2 - |\phi_\xi(x + x_0 + v_gt)|^2 \right]^{2} & x < 0, \\
|t|^2 \int_{-\infty}^{t} du e^{i\Omega(t-u)} \sum_{k,x_0} e^{-\int_{u}^{t} d\tau \gamma_1(k,x_0,\tau)} \gamma_1(k,x_0,\tau) \times \\
|\phi_\xi(x + x_0 - v_gt)|^2 & x > 0. \end{cases}
\]

At zero frequency, \( Y(x,\Omega = 0) = G \) is purely real and uniform throughout the lead region.
Therefore, the d.c. limit, Eq.(7), is correctly recovered. At a finite frequency \( \Omega \) comparable
to \( \Omega_0 = 2\pi/\tau \) (\( \tau = L/v_g + \hbar d\delta_c/dE \) with \( v_g = \hbar k_F/m \)), \( y \) becomes complex and oscillates
with \( x \). These behaviors are illustrated in Fig. 4. In Fig. 5, we plot the average admittance
\( \bar{Y}(\Omega) = \frac{1}{T} \int_{-L/2}^{L/2} dx Y(x,\Omega) \) versus \( \Omega/\Omega_0 \). As the frequency increases, the system becomes
less and less capable to follow the change of \( u(t) \). This results in the decrease of admittance
amplitude and the retardation in response leads to an inductive behavior as demonstrated
by the imaginary part of the admittance. The characteristic frequency \( \Omega_0 \) typically lies in
the range between a few GHz and a few THz. Although directly probing the spatial variance
of current within the channel poses considerable challenge, the experimental measurements of the average admittance for frequencies up to several THz are certainly feasible. Similar experiments have actually been carried out for the double barrier structures. [5]

**IV. SUMMARY**

In summary, we have presented a time dependent Landauer approach to study the dynamic response of a mesoscopic conductor and demonstrated the frequency dependence of its admittance. The relevant time scale is found to be \( \tau = L/v_g + \tau_s \) which leads to a characteristic frequency \( \Omega_0 = 2\pi/\tau \). When the bias frequency exceeds \( \Omega_0 \), the average admittance falls below its \( dc \) limit and spatial variations start to appear on a scale \( l_\Omega = v_g/\Omega \).

Although only a two-probe model is fully investigated in the present work, the conclusion we draw here should also hold for the general multi-probe systems. Moreover, this work can also be generalized to include the displacement current which, at high frequencies, may yield a contribution to the total measured circuit current comparable to the intrinsic part fully discussed here. Finally, the present study is based on the assumption that the reservoirs respond to the signal electric field immediately. This naturally imposes an upper restriction on the frequency given by the inverse of the reservoir dissipation time which is typically around 10THz.

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REFERENCES


[16] The spreading of the wavepacket in the lead region is neglected.

FIG. 1. A three-probe mesoscopic system consists of three reservoirs at chemical potential $\mu_1$, $\mu_2$, and $\mu_3$ and three perfect conducting leads that connect the scatterer with the reservoirs.
FIG. 2. A two-probe mesoscopic system.
FIG. 3. The coupling strength of a wave packet to the reservoirs $\gamma_1(k, x_0, t)$ (solid curve, scaled by $\gamma$) and $\gamma_2(k, x_0, t)$ (dashed curve, scaled by $\gamma$) vs the center of the packet $\left( v_k t - x_0 \right)$ (scaled by $\xi$). The conductor length $L = 10\xi$. 
FIG. 4. The relative admittance $y(x, \Omega)$ as a function of spatial coordinate $x/L$ for $\Omega = \Omega_0$ (the solid curve is its real part and the dashed curve is its imaginary part).
FIG. 5. Average relative admittance $\tilde{Y}(\Omega)/G$ vs frequency $\Omega/\Omega_0$. The solid curve is the real part (conductance) and the dashed line is the imaginary part.